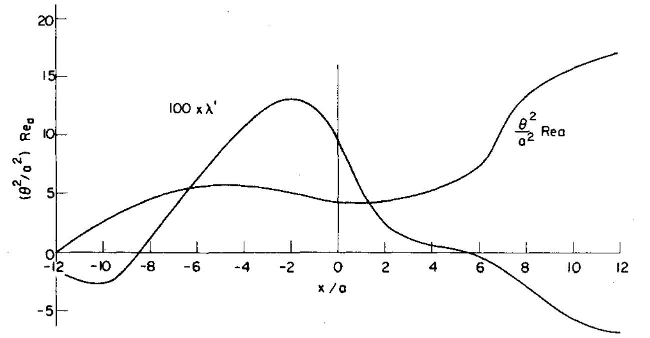

Fig. 1 Distribution of (Θ^2/a^2) Re_a and μ'—source in free air; $\psi(-a) = -4U_a a$.

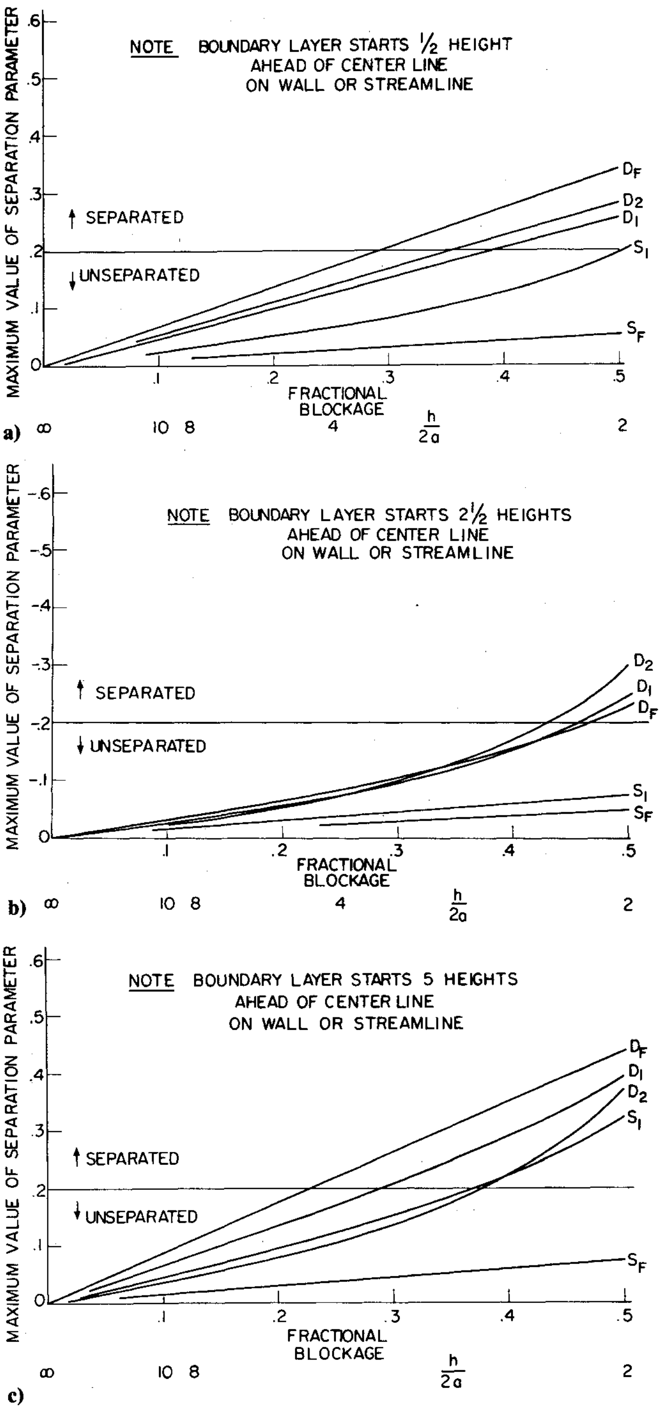

Fig. 2 Boundary layer starts a) ½ height ahead of centerline on wall or streamline, b) 2½ heights ahead of centerline on wall or streamline, and c) 5 heights ahead of centerline on wall or streamline.

Table 2 Initial value of $(\Theta^2/a^2)Re_a$ for upstream separation

	Source	Source + 1 wall	Doublet	Doublet + 1 wall	Doublet + 2 walls
−2	24	3.4	66	54	7150
−4	29	9.6	7120	400	Not computed
−8	93	156	−	37.6	Not computed

Note typical blocking in classical wind tunnels is of the order of 5% or less. In that case separation on the wall of the type described here is unlikely to happen. However, use of large models and self-streamlining walls, particularly near stall where gradients could be numerically large, may present problems.

Alternately, in confined flowfields, as may be encountered between an external store and its rack, it is possible that induced separation of this type could exist. This is particularly likely because the store usually extends well in front of the rack and so a relatively thick boundary layer could be expected. Note that it is possible in a confined flow for the aft separation (as in the source plus one wall) to choke or block the flow and to cause the separation point to move well forward. Flow between a wall and any boattail shape generates a deceleration that causes the wall boundary layer to be particularly susceptible to the latter type of induced separation. Finally, below a critical Mach number the results suggest likelihood of separation may be reduced through the use of aerodynamic interference to accelerate the flow locally.

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Sound Radiation From Ducts: A Comparison of Admittance Values

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WHEN considering the radiation from an open duct, it is found that some of the energy is radiated and some reflected (with a phase shift) back down the duct. It is

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common to associate an effective admittance (impedance or reflection coefficient) at the exit plane with this phenomenon. In this Note, three methods for obtaining an effective admittance are compared.

The first method follows the analyses of Helmholtz and Rayleigh in which the end of the duct is approximated by a piston radiating into a half-space from an infinite baffle. In this analysis, the classical integral representation of the solutions of the Helmholtz equation is solved with certain approximations. Results for this configuration, commonly known as a flanged pipe, using this method are presented in Ref. 1.

The second method consists of the solution of a Weiner-Hopf type of integral equation. In this analysis, the duct is assumed to be semi-infinite in length and infinitely thin. Results for this type of analysis are presented in Ref. 2. This configuration is commonly known as the unflanged pipe.

It is interesting to note that these two configurations represent the logical limits of this type of problem in that the first can be viewed as an infinitely thick duct while the second is infinitely thin. Neither of these configurations, however, can account for the case of a duct of finite length.

The third method employs a special cylindrically symmetric integral representation of the exterior solutions of the Helmholtz equation.[3-5] Using this method it is possible to calculate the acoustic pressure and velocity anywhere in the external field—including the inside of the duct itself. From these values inside the duct, an effective admittance can be calculated using a simple standing wave analysis like that used in an impedance tube. All that is required for this method is knowledge of two complex acoustic quantities—amplitude and phase—at two points in the duct.

The geometric restrictions on the integral technique are that the duct or any radiating body must be a finite (i.e., no infinitely thin walls), closed body. A sketch of the duct used in this analysis is presented in Fig. 1.

To determine if the length of the duct makes a significant difference in the admittance values at the duct entrance, computer analyses of ducts of varying lengths were run ($L/a = 1,2,3$) at two different nondimensional wave numbers ($ka = 1$ and 3). Since the method is capable of calculating the actual radially varying admittance across the exit plane of the duct, these are presented in Fig. 2. The driver consisted of specifying a unit normal acoustic velocity, while on the rest of the body the admittance was specified as zero. As can be seen, the length of the duct L has little effect on the admittance, defined as the ratio of the component of the acoustic velocity normal to the surface to the acoustic pressure. Also noted for the sake of comparison are the values for the flanged and unflanged pipe at the appropriate values of ka.

Computer analyses of a duct with $L/a = 3$ were then done with the same boundary conditions specified previously for various values of ka. For each case, the acoustic potential and velocity were calculated at 11 equally spaced points along the centerline of the duct from $Z = 1$ to $Z = 2$. A standing wave analysis was then done employing a least-square method to solve the overdetermined system of equations. The results of these analyses are presented in Fig. 3 along with the values for a flanged and an unflanged pipe. Similar analyses were done off the centerline of the duct; however, no significant differences in the computed values for the admittance at the entrance plane of the duct were found. Also noted for comparison in Fig. 3 are the values of the admittance calculated on the centerline of the duct at the exit plane.

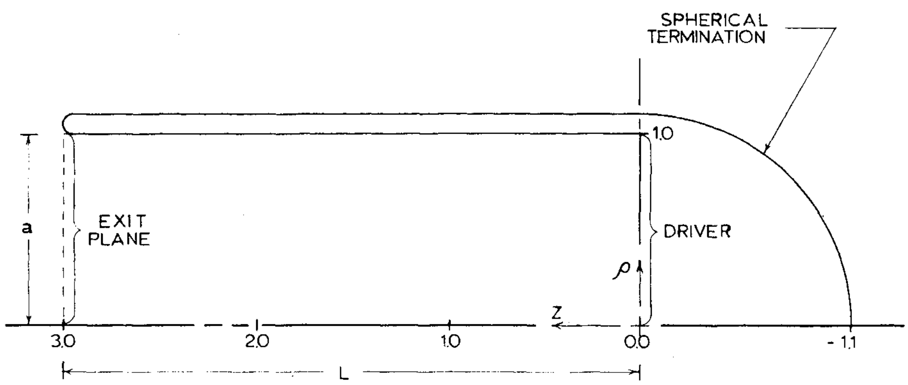

Fig. 1 Straight duct geometry.

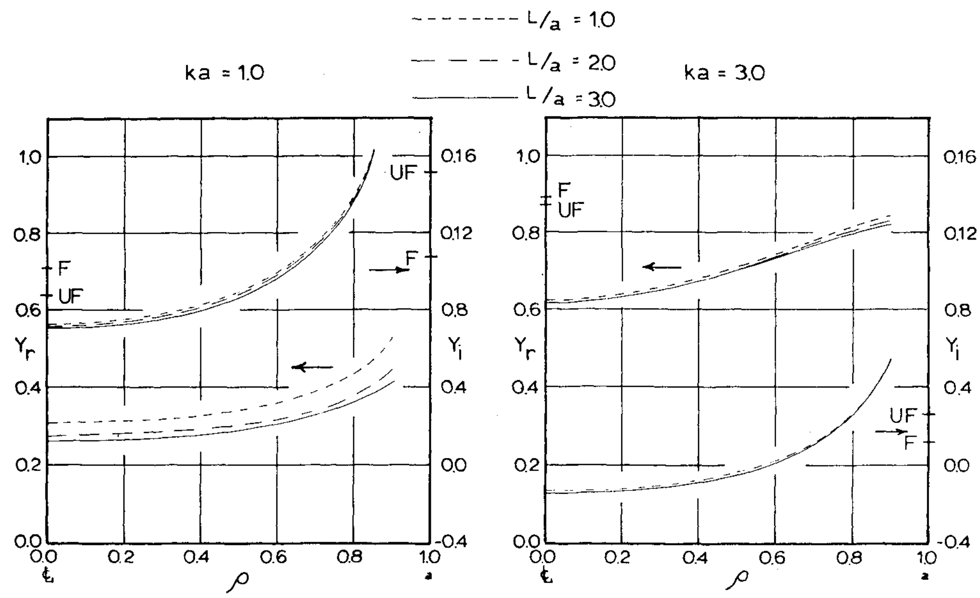

Fig. 2 Admittance at the exit plane of straight ducts.

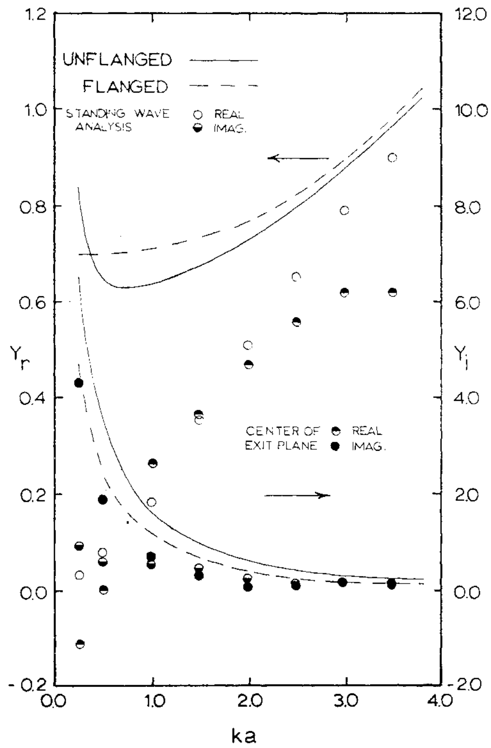

Fig. 3 Admittance at the open end of a straight duct.

Acknowledgments

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Further Observations on the Strained Coordinate Method for Transonic Flows

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Introduction

ONE of the features of a singular perturbation problem is the fact that higher approximations are more singular than the first approximation. This is due to the breakdown, in some region, of the basic assumption of small perturbations. In other words, a perturbation solution is not uniformly valid since in some region of the flow the basic assumptions do not apply. In certain classes of problems, a perturbation solution can be made uniformly valid by use of the method of matched asymptotic expansions[1] or by the method of strained coordinates.[2] In the latter method, the source of the singularity is removed by straining the independent coordinates such that the singularity apparent in the first approximation is not compounded in higher approximations. This coordinate straining is fairly arbitrary, since a number of functions can be devised that remove the compounding of the singularity.

In the last few years, a development of the strained coordinate method, as applied to transonic flow problems, has appeared in the literature.[3,4] In this method, the magnitude of the straining is determined by physical conditions (namely, that the shock wave always moves to its correct location), but there is still a considerable degree of arbitrariness concerning the choice of a straining function. In Ref. 4, an attempt is made to determine the effect of altering the form of the straining function by using two different straining functions in the computation of the same example. No discernible difference between the results was detected. However, it is desirable to prove analytically the dependence (or lack thereof) of the final pressure distribution on a particular straining function, and it is this problem that is considered here. It is found that the pressure distribution is independent of the straining function provided this function moves the shock location to the correct position.

Analysis

In a Cartesian coordinate system (x,z) with a streamwise velocity component $U(x,z)$, the total velocity for a perturbation of magnitude ϵ is given by[3]

$$U(x,z)=U_0(x',z)-\epsilon U_0(x',z)\,\delta x_s x_{1_{x'}}(x')+\epsilon U_1(x',z) \quad (1)$$

where the strained coordinate x' is related to the physical coordinate by

$$x=x'+\epsilon\delta x_s x_1(x') \quad (2)$$

and $x_1(x')$ is the straining function. Also, $U_0(x',z)=\phi_{0_{x'}}(x',z)$, and $\phi_0(x',z)$ is given by the solution of the equation

$$\phi_{0_{x'x'}}+\phi_{0_{zz}}=\phi_{0_{x'}}\phi_{0'_{x'x'}} \quad (3)$$

with the boundary condition

$$\phi_{0_z}(x',\pm 0)=\pm z_{0_{x'}}(x') \quad (4)$$

The term $U_1(x',z)=\phi_{1_{x'}}(x',z)$, and $\phi_1(x',z)$ is given by the solution of the equation

$$\phi_{1_{x'x'}}+\phi_{1_{zz}}=\left(\phi_{1_{x'}}\phi_{0_{x'}}\right)_{x'}+\delta x_s\left[x_{1_{x'}}\left(\phi_{0_{x'}}-\phi^2_{0_{x'}}\right)\right]_{x'} + \delta x_s x_{1_{x'}}\left(\phi_{0_{x'}}-\tfrac{1}{2}\phi^2_{0_{x'}}\right)_{x'} \quad (5)$$

with the boundary condition

$$\phi_{1_z}(x',\pm 0)=\pm z_1'(x')\pm\delta x_s x_1(x')\,z_0''(x') \quad (6)$$

In Eqs. (2),(5), and (6), $\epsilon\delta x_s$ is the shock movement.

In order to examine properly the behavior of the perturbation solution, it is necessary to write the equations in a common set of independent variables. On either side of the shock wave, $U(x,z)$ is piecewise continuous, and hence can be expanded in a Taylor series. Thus,

$$U(x',z)=U(x,z)-\epsilon\delta x_s x_1(x')\,U_x(x,z) \quad (7)$$

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